| **Big Idea** | **K-2** | **3-6** | **7-8** | **9-10** | **11-12** |
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| BI1 A number tells how much or how many. | • There are many ways to count. [*This could lead to a discussion of skip counting which is a precursor to proportionality.*] – BI1  • You can represent a number in a variety of ways. Each representation can focus on a different aspect of the number. *[This could lead into a discussion of doubles and maybe even doubles strategies.] – BI3*  • A fraction is not meaningful without knowing the size of the whole. [*1/2 can look really big or really little—it’s all about the relationship of the part to the whole.*] – BI1 |  | • The size of a fraction should be thought of as the relationship between its numerator and denominator. [*At this level, we want kids to realize ½ = 2/4 not because they take the same space in a pie, but because 2 is twice 1 just like 4 is twice 2.*] – BI4, (BI1) |  |  |
| BI 2  Classifying numbers or numerical relationships provides information about the characteristics of the numbers or the relationship.  *<For example, knowing that 6 is 2/3 of 9 (and therefore that the ratio 6:9 is equivalent to 2:3) tells us that if the ratio of girls to boys in a group is 6:9, there can be 10 people in the group but there could not be 11.>* |  | • To count the number in a group, we often create subgroups and count the number of subgroups. *[This leads into a discussion of place value where we think of a number in terms of how many tens or hundreds; it also relates to skip counting and to early multiplication.] –* BI2, BI3, (BI5)  • Classifying numbers as factors and/or multiple of other numbers provides additional information about those numbers. [e.g. *If I classify a number as a multiple of 6, I know lots about it. Multiplication is, of course, the basis for proportional thinking.]* – BI2, BI5  • Renaming fractions is often the key to comparing them or computing with them. [*For example, it is easier to compare ½ and 5/8 if you think of ½ as 4/8. You use the proportion 1/ 2 = 4/8 to do the comparison.*] – BI2, BI3, BI4 | • Classifying numbers as factors and/or multiple of other numbers provides additional information about those numbers. [e.g. *If I classify a number as a multiple of 6, I know lots about it.] – BI2* | • Classifying numbers as factors and/or multiple of other numbers provides additional information about those numbers. [e.g. *If I know that 28 is a multiple of 7, it helps me solve the proportion: 28:x = 7:56.] – BI2*  •An important way to relate two variables is to consider rate of change, i.e. how one variable changes when the other changes. [*This leads to the definition of linear relations as ones with a constant rate of change. When we say two variables are proportional, we are talking about linear relations.*] – BI2  • There are two fundamentally different kinds of linear relations- in one situation, the values of one value are proportional to the corresponding values of the other and in the other situation, a constant must be subtracted off the dependent variable before its values are proportional to the corresponding values of the independent variable. [*This helps distinguish partial and direct variation in terms of whether, if one doubles the x, the y is doubled or not.*] – BI2  • Because the linear dimensions of similar shapes are proportional, one can determine the dimensions of one shape by knowing the dimensions of the other and the proportion factor. [*This, of course, leads into all work on trig.*] – BI2 | • Classifying numbers as factors and/or multiple of other numbers provides additional information about those numbers. [e.g. *If I know that 28 is a multiple of 7, it helps me solve the proportion: 28:x = 7:56.] – BI2*  • The degree of a polynomial is related to the finite difference where the differences are proportional to the values of the independent variable or proportional to the values of the independent variable if a constant is subtracted.[*A quadratic relation is one where the first differences form a linear relation so that the values of those differences are directly proportional to or the values of those differences minus a constant are directly proportional to the values of the independent variable.] – BI2*  • If the general term of an arithmetic sequence is represented as a + (n-1)d, then the values of (tn-a) are proportional to n. – BI2  • Variables can be inversely proportional if the product of their corresponding values is constant. [*For example, the time taken to complete a job varies inversely with the number of laborers since 2 laborers doing 1/2 of the job accomplish the same as 3 doing 1/3 or 4 doing 1/ 4, etc.] – BI2* |
| BI 3  There are many equivalent representations for a number or numerical relationship. Each representation may emphasize something different about that number or relationship.  *<For example, one can represent the ratio 2:5 as 4:10 or 6:15 or 2/5 or 40% or whatever might be more convenient for a particular situation.>* | • To count the number in a group, we often create subgroups and count the number of subgroups. *[This leads into a discussion of place value where we think of a number in terms of how many tens or hundreds. This relates to proportionality in that we are thinking of 2 hundreds as proportional to 2 tens or 2 ones—only the unit changed.] – BI3, BI5* | • To count the number in a group, we often create subgroups and count the number of subgroups. *[This leads into a discussion of place value where we think of a number in terms of how many tens or hundreds; it also relates to skip counting and to early multiplication.] –* BI2, BI3, (BI5)  • If you multiply any factor of a number by n, you multiply the product by n. [*For example, this leads later to the idea that if a is proportional to b, then if you double a, you double b.] –* BI3, BI5  • To divide two numbers, you can multiply or divide both by the same amount without changing the quotient. [*For example, this leads later to the idea that if na is proportional to nb, then a is proportional to b.*] – BI3, BI5  • You can describe the same portion using an infinite number of fractions. [*This leads into the notion of equivalence and eventually to a definition of proportion.*] – BI3, BI4  •Thinking of numbers as factors or multiples of other numbers provides alternative representations of those numbers. *[This could lead into representing products as arrays, and other definitions of and representations of multiplication. Multiplication is, of course, the basis for proportional thinking.]* – BI3, BI5  • There are relationships between all four operations. In particular, multiplication and division are opposites. [*This leads into a discussion of the notion that if 8 is four 2s, then 2 is ¼ of an 8. When we think of a ratio, we can think of the ratio either way.*] – BI3, BI5  • Renaming fractions is often the key to comparing them or computing with them. [*For example, it is easier to compare ½ and 5/8 if you think of ½ as 4/8. You use the proportion 1/ 2 = 4/8 to do the comparison.*] – BI2, BI3, BI4 | • If you multiply any factor of a number by n, you multiply the product by n. [*For example, if a is proportional to b, then if you double a, you double b. This begins discussions of rate of change.] – BI3, BI5*  • To divide two numbers, you can multiply or divide both by the same amount without changing the quotient. [*For example, if na is proportional to nb, then a is proportional to b.*] – BI3, BI5  • You can describe the same portion using an infinite number of fractions. [*This leads into the notion of equivalence and eventually to a definition of proportion.*] – BI3  • Renaming fractions or ratios is often the key to comparing them or computing with them. [*For example, it is easier to compare ½ and 5/8 if you think of ½ as 4/8; it is easier to compare 2:5 and 3:7 if we write both as percents. This ties to the more general relationships between fractions, decimals and percents and the notion that we use proportions to find the “friendlier” names.*] – BI3, BI4 | • You can describe the same fraction in an infinite number of ways. [*This leads into a definition of proportion.*] – BI3  • One can think of a proportion as a numerical transformation. [*e.g. To determine the value of x if 3:x = 2:15, you think of the “ transformation” 2/3 being applied to x or 3/2 being applied to 15.] – BI3* | • If you multiply any factor of a number by n, you multiply the product by n. [*For example, if a is proportional to b, then if you double a, you double b. This involves rate of change.] – BI3*  • To divide two numbers, you can multiply or divide both by the same amount without changing the quotient. [*For example, if na is proportional to nb, then a is proportional to b.*] – BI3, BI5  • Because the linear dimensions of similar shapes are proportional, one can determine the dimensions of one shape by knowing the dimensions of the other and the proportion factor. [*This, of course, leads into all work on trig.*] – BI3, BI4  •Exploring trigonometric situations is facilitated by the ability to solve proportions. *[e.g. if tan a = 0.5 and the opposite side to angle a is 12, then solving the proportion 1/ 2 = 12/x helps you determine other information about angle a.] – BI3, BI5*  Exploring probability *– BI3, BI5*  • Certain transformations of graphs can be thought of as creating proportional values, but other transformations do not. [*For example, the values of y = 2x2 are proportional to corresponding values of y = x2, but the values of y = (x - 2)2 are not proportional to corresponding values of y = x2. ] – BI3* |
| BI 4  Numbers are compared in many ways. Sometimes they are compared to each other. Other times, they are compared to benchmark numbers.  *<For example, one way to compare 63 to 21 is to say that 63 is 42 more, but a different way is to say that 63 is 3 times as much. One way to think of the ratio* *18:37 is to think of this as almost ½.>* | • To compare the numbers of items in two sets, you can match the items, one to one, and see which has more or you can compare the position of the numbers that describe the two amounts in the number sequence. *[This could lead into a discussion of comparisons—later we can compare 6 and 3 by thinking of doubling but initially we think of 6 as 3 more.] – BI4*  • Students gain a sense of the size of numbers by comparing them to familiar benchmark numbers. *[Later, this leads into the idea that I compare in terms of how many of a number there are, e.g. 27 is around 5 groups of five.] – BI4* | • You can describe the same portion using an infinite number of fractions. [*This leads into the notion of equivalence and eventually to a definition of proportion.*] – BI3, BI4  • To compare the numbers of items in two sets, you can use a ratio that focuses on how many of “a” for every “b”. *[The intention is to focus on multiplicative comparisons.] – BI4, BI5*  • Renaming fractions is often the key to comparing them or computing with them. [*For example, it is easier to compare ½ and 5/8 if you think of ½ as 4/8. You use the proportion 1/ 2 = 4/8 to do the comparison.*] – BI2, BI3, BI4 | • To compare the numbers of items in two sets, you can use a ratio that focuses on how many of “a” for every “b”. *[The intention is to focus on multiplicative comparisons, which is what ratios do.] – BI4*  • The size of a fraction should be thought of as the relationship between its numerator and denominator. [*At this level, we want kids to realize ½ = 2/4 not because they take the same space in a pie, but because 2 is twice 1 just like 4 is twice 2.*] – BI4, (BI1)  • Renaming fractions or ratios is often the key to comparing them or computing with them. [*For example, it is easier to compare ½ and 5/8 if you think of ½ as 4/8; it is easier to compare 2:5 and 3:7 if we write both as percents. This ties to the more general relationships between fractions, decimals and percents and the notion that we use proportions to find the “friendlier” names.*] – BI3, BI4 | • The size of a fraction should be thought of as the relationship between its numerator and denominator. [*At this level, we want kids to realize ½ = 2/4 not because they take the same space in a pie, but because 2 is twice 1 just like 4 is twice 2.*] – BI4  • Renaming fractions or ratios is often the key to comparing them or computing with them. [*For example, it is easier to compare ½ and 5/8 if you think of ½ as 4/8; it is easier to compare 2:5 and 3:7 if we write both as percents. This ties to the more general relationships between fractions, decimals and percents.*] – BI4  • If you multiply any factor of a number by n, you multiply the product by n. [*For example, if a is proportional to b, then if you double a, you double b. This begins discussions of rate of change.] – BI4, BI5*  • To divide two numbers, you can multiply or divide both by the same amount without changing the quotient. [*For example, if na is proportional to nb, then a is proportional to b.*] – BI4, BI5 | •An important way to relate two variables is to consider rate of change, i.e. how one variable changes when the other changes. [*This leads to the definition of linear relations as ones with a constant rate of change and non-linear relations as ones where the rate of change is not constant.*] – BI4  • Because the linear dimensions of similar shapes are proportional, one can determine the dimensions of one shape by knowing the dimensions of the other and the proportion factor. [*This, of course, leads into all work on trig.*] – BI3, BI4 |
| BI 5  The operations of addition, subtraction, multiplication and division hold the same fundamental meaning no matter the domain to which they are applied.  *<For example, a) proportions relate to multiplicative comparisons—one of the meanings of multiplication is a comparison meaning, e.g. 4 x means the ratio 4:1. b) if we think of a comparison where one number is a multiple of another, we know the second number is also a multiple of the first (the reciprocal). c) We can also think about ratios with respect to “scalar” multiplication; for example, a mixture that is 9 parts sugar: 1 part water is “3 times as sweet” as one that is 3 parts sugar: 1 part water. And more…>* | • To count the number in a group, we often create subgroups and count the number of subgroups. *[This leads into a discussion of place value where we think of a number in terms of how many tens or hundreds. This relates to proportionality in that we are thinking of 2 hundreds as proportional to 2 tens or 2 ones—only the unit changed.] – BI3, BI5* | • If you multiply any factor of a number by n, you multiply the product by n. [*For example, this leads later to the idea that if a is proportional to b, then if you double a, you double b.] –* BI3, BI5  • To divide two numbers, you can multiply or divide both by the same amount without changing the quotient. [*For example, this leads later to the idea that if na is proportional to nb, then a is proportional to b.*] – BI3, BI5  •Thinking of numbers as factors or multiples of other numbers provides alternative representations of those numbers. *[This could lead into representing products as arrays, and other definitions of and representations of multiplication. Multiplication is, of course, the basis for proportional thinking.]* – BI3, BI5  • To count the number in a group, we often create subgroups and count the number of subgroups. *[This leads into a discussion of place value where we think of a number in terms of how many tens or hundreds; it also relates to skip counting and to early multiplication.] –* BI2, BI3, (BI5)  • To compare the numbers of items in two sets, you can use a ratio that focuses on how many of “a” for every “b”. *[The intention is to focus on multiplicative comparisons.] – BI4, BI5*  • There are relationships between all four operations. In particular, multiplication and division are opposites. [*This leads into a discussion of the notion that if 8 is four 2s, then 2 is ¼ of an 8. When we think of a ratio, we can think of the ratio either way.*] – BI3, BI5 | • If you multiply any factor of a number by n, you multiply the product by n. [*For example, if a is proportional to b, then if you double a, you double b. This begins discussions of rate of change.] – BI3, BI5*  • To divide two numbers, you can multiply or divide both by the same amount without changing the quotient. [*For example, if na is proportional to nb, then a is proportional to b.*] – BI3, BI5  • There are relationships between all four operations. In particular, multiplication and division are opposites. [*This leads into a discussion of the notion that if 8 is four 2s, then 2 is ¼ of an 8. When thinking proportionally, we want to relate the numbers in a multiplicative way, but flexibly.*] – BI5 | • If you multiply any factor of a number by n, you multiply the product by n. [*For example, if a is proportional to b, then if you double a, you double b. This begins discussions of rate of change.] – BI4, BI5*  • To divide two numbers, you can multiply or divide both by the same amount without changing the quotient. [*For example, if na is proportional to nb, then a is proportional to b.*] – BI4, BI5 | • To divide two numbers, you can multiply or divide both by the same amount without changing the quotient. [*For example, if na is proportional to nb, then a is proportional to b.*] – BI3, BI5  •Exploring trigonometric situations is facilitated by the ability to solve proportions. *[e.g. if tan a = 0.5 and the opposite side to angle a is 12, then solving the proportion 1/ 2 = 12/x helps you determine other information about angle a.] – BI3, BI5*  Exploring probability *– BI3, BI5*  • Measurement conversions involving units of the same attribute involve proportional thinking, but measurement conversions involving units of different attributes usually do not involve proportional thinking. – BI5  • Simple interest situations involve proportions based on any of amount invested, time and interest rate, but compound interest situations do not. – BI5 |